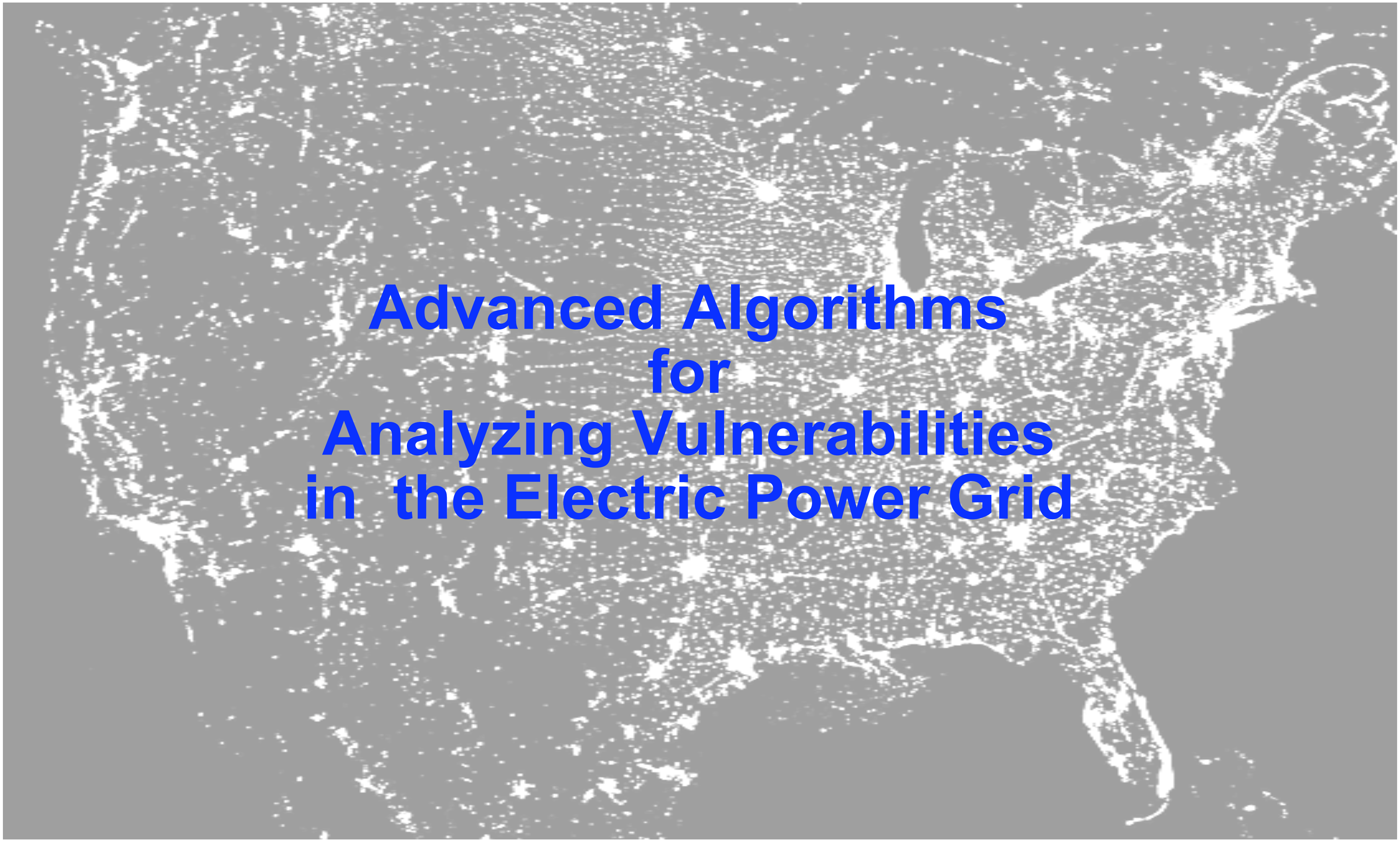

Advanced Algorithms for Analyzing Vulnerabilities in the Electric Power Grid

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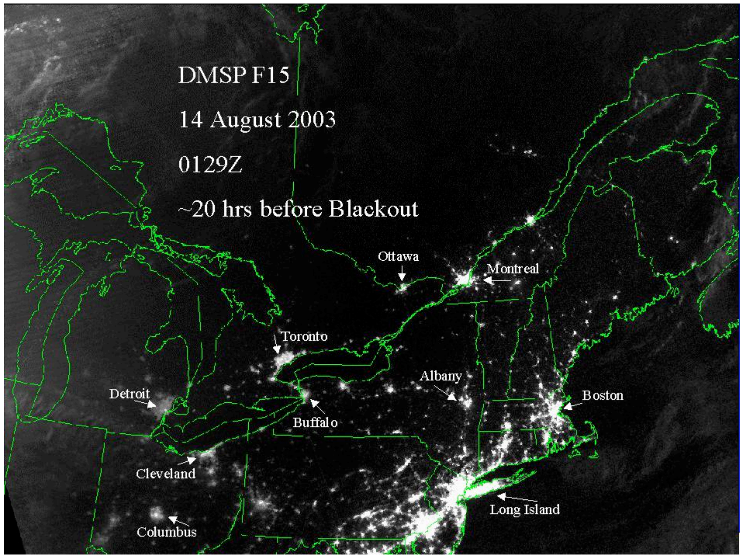
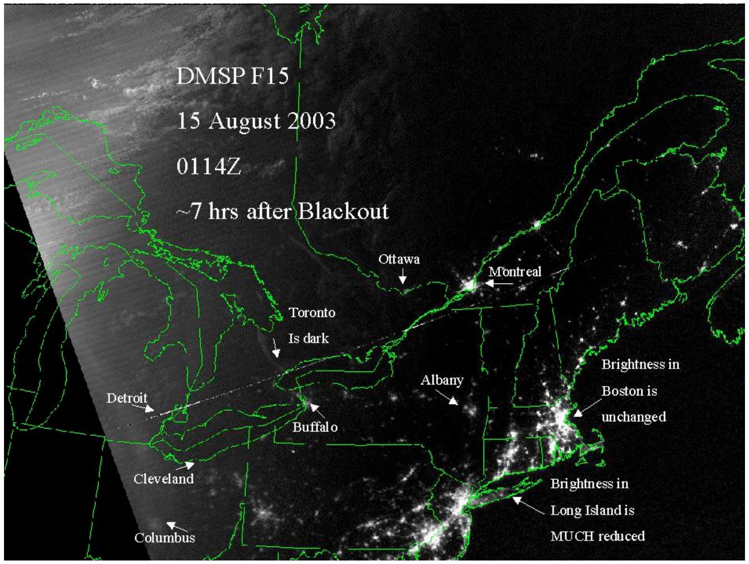
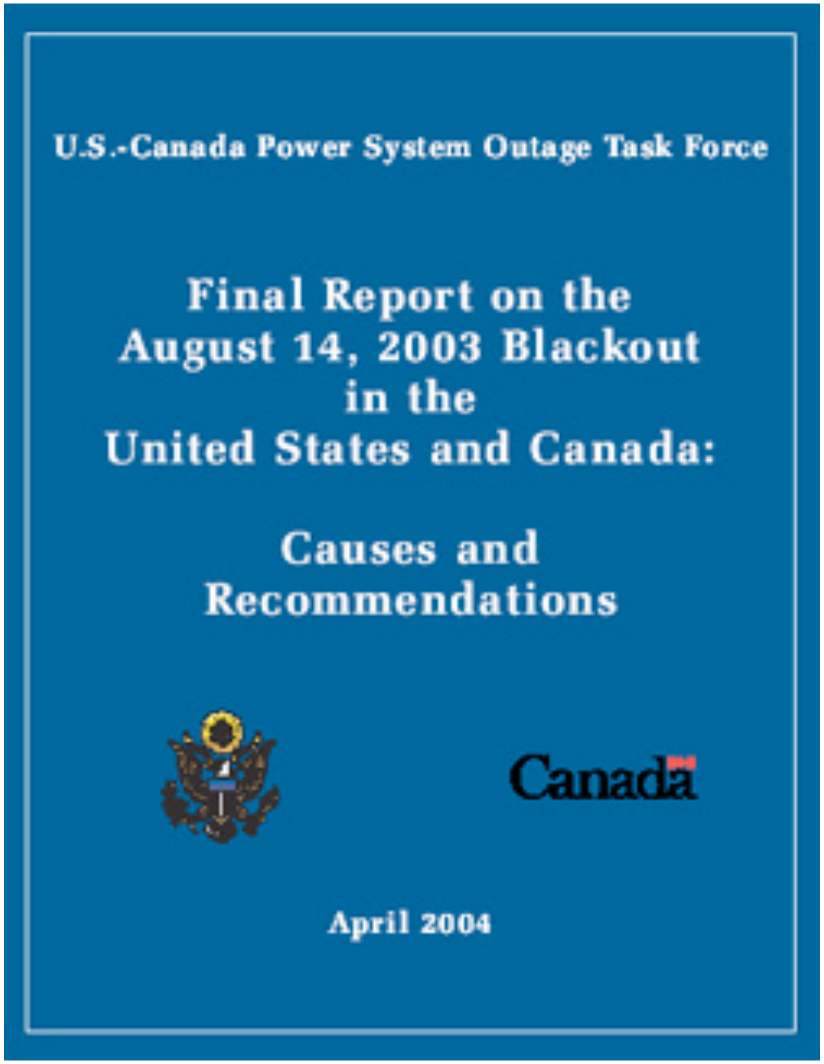
Power blackouts are a global problem

- August 2003 blackout affected 50 million people in New York, Pennsylvania, Ohio, Michigan, Vermont, Massachusetts, Connecticut, New Jersey, Ontario.
- The time to recover from the blackout was as long as 4 days at an estimated cost of $4-10 B
- Similar occurrences elsewhere: Brazil (1999), France-Switzerland-Italy (2003)



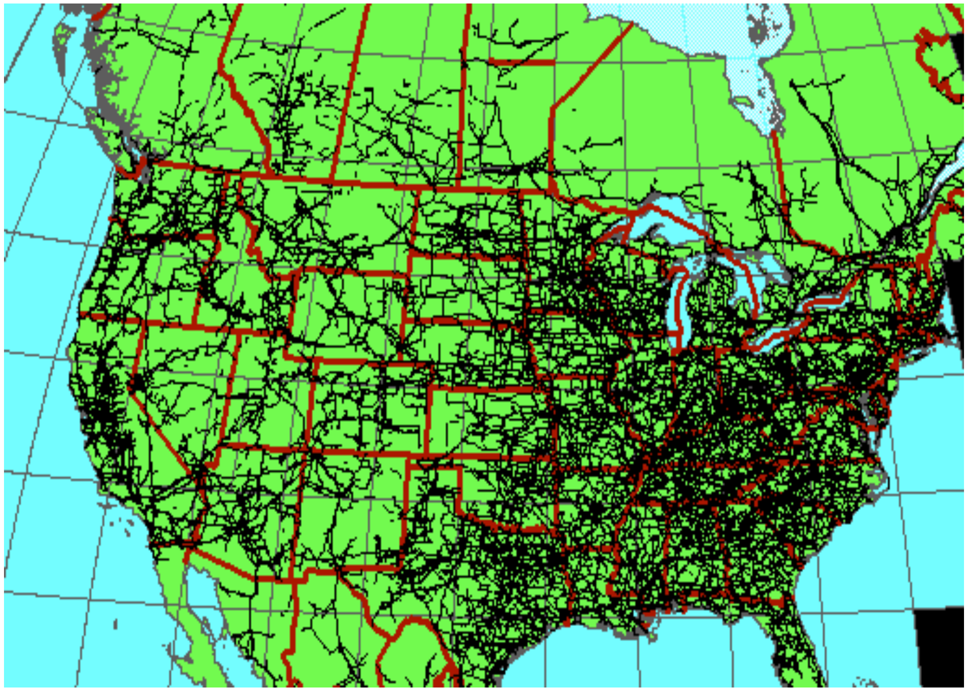


Investigation of Aug. 14, 2003 Blackout



- Consortium for Electric Reliability Technology Solutions (CERTS) coordinated/staffed initial fact-finding field investigations
- J. Eto (LBNL) appointed to Electric Systems Working Group
  - Organized/conducted technical workshops
  - Staffed technical analysis teams (Root Cause, Frequency, Data Warehousing)
- *Recommendation 13:*
  - *DOE should expand its research programs on reliability related tools and technologies*

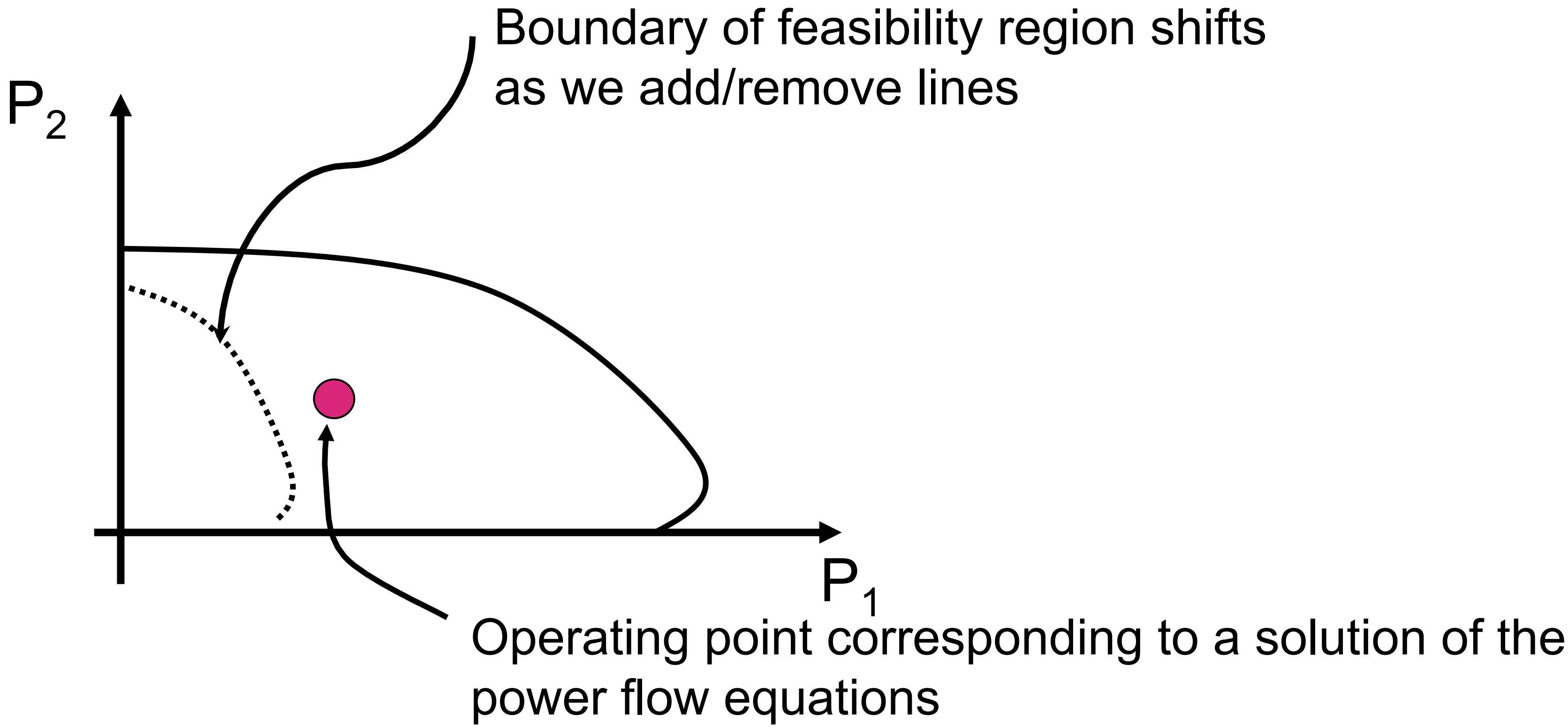
The power grid is increasingly vulnerable as the complexity of the system grows



Northeast blackout started with three broken lines.

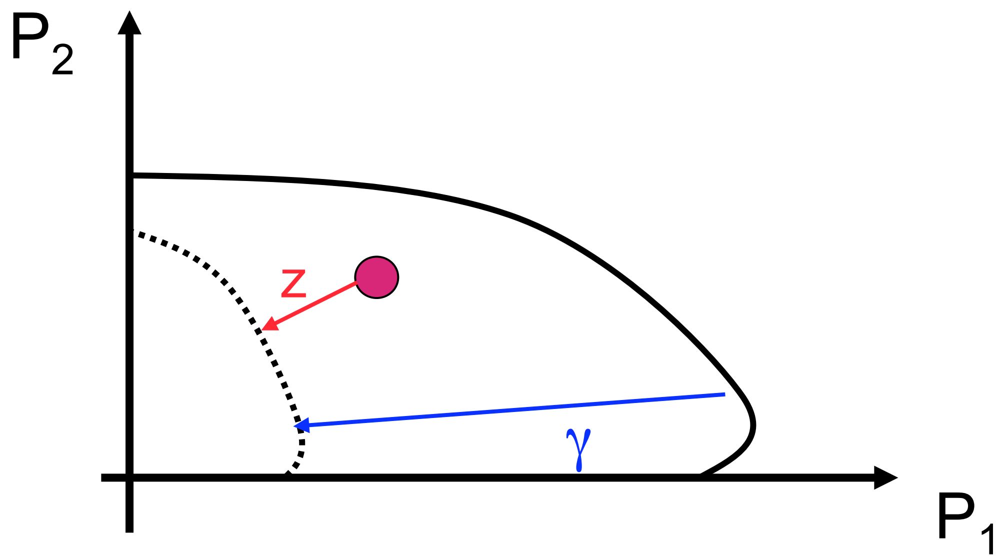
- Problem: the current standard requires the system to be resilient to only one failure, because higher standards are not enforceable.
- Goal: develop computational methods that
  - detect vulnerabilities of the power network
  - determine how to update the system to increase security
  - scale and are widely applicable
- Challenge: requires combinatorial and nonlinear optimization
  - NP-hard
  - large-scale problems

Graphical representation of a blackout



- Blackout corresponds to infeasibility of power flow equations.
- Cascading is initiated by a significant disturbance to the system.
- Our focus is detecting initiating events and analyzing the network for vulnerabilities.

Vulnerability analysis viewed as a bi-level optimization problem



- Add integer (binary) line parameters, γ, to identify broken lines
- Measure the blackout severity as the distance to feasibility boundary
- Goal:
  - cut minimum number of lines so that
  - the shortest distance to feasibility (i.e. severity) is at least as large as a specified target

This approach leads to a Mixed Integer Nonlinear Program (MINLP)

$$\min_{\lambda,z,\theta,\mu_1,\mu_2,\mu_3,\mu_4,\mu_5,\mu_6} |\lambda|$$

minimize number of lines cut

s.t.

$$F(AD(1-\gamma),\theta,p+z)=0$$
$$-\pi/2 \le AD(1-\gamma)\theta \le \pi/2$$

feasible power flow

$$-e^T z_g \ge S$$

severity above threshold

$$0 \le p_g + z_g \le p_g$$
$$p_l \le p_l + z_l \le 0$$

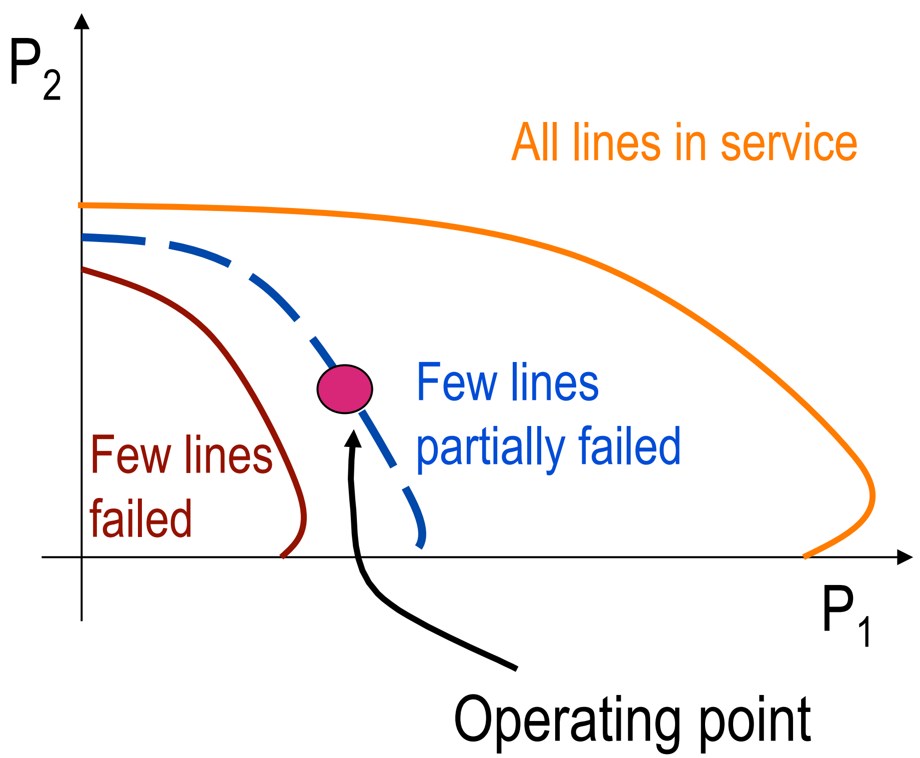
feasible load shedding

$$\begin{pmatrix}-e\\0\end{pmatrix}-\begin{pmatrix}\lambda_g\\\lambda_l\end{pmatrix}+\begin{pmatrix}\mu_4-\mu_3\\\mu_2-\mu_1\end{pmatrix}=0$$
$$\lambda^T\frac{\partial F}{\partial\theta}+A^T D(1-\gamma)(\mu_6-\mu_5)=0$$
$$\mu_1 z_l=0;\quad \mu_2(p_l+z_l)=0$$
$$\mu_4 z_g=0;\quad \mu_3(p_g+z_g)=0;$$
$$\mu_5(\pi/2+AD(1-\gamma)\theta)=0;$$
$$\mu_6(\pi/2-AD(1-\gamma)\theta)=0;$$
$$\mu_1,\ldots,\mu_6\ge 0$$

*s*atisfy the KKT optimality conditions

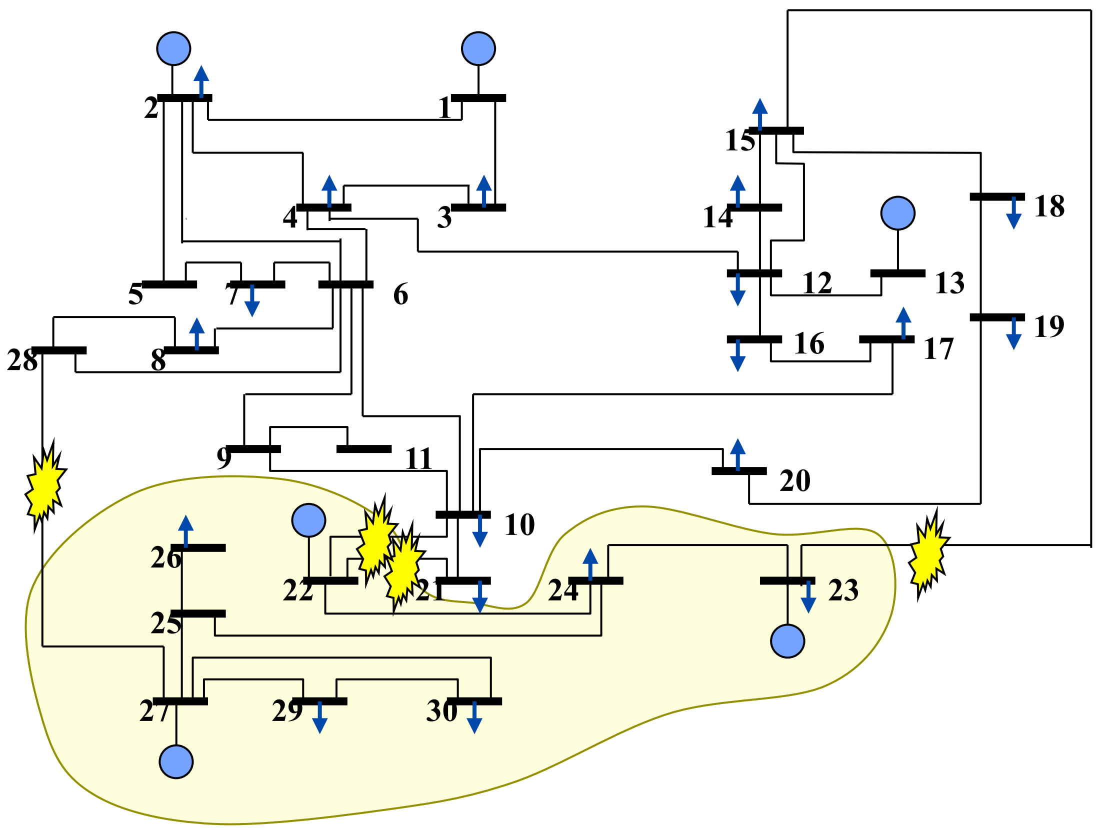
$$\gamma\in\{0,1\}$$

Relaxed model - pictorially



- Feasibility boundary moves as lines fail.
- Relaxed model: lines "partially" fail.
- Benefits:
  - Operating point now lies exactly on the feasibility boundary
  - Transforms the mixed-integer problem (difficult) into a continuous one (easier).

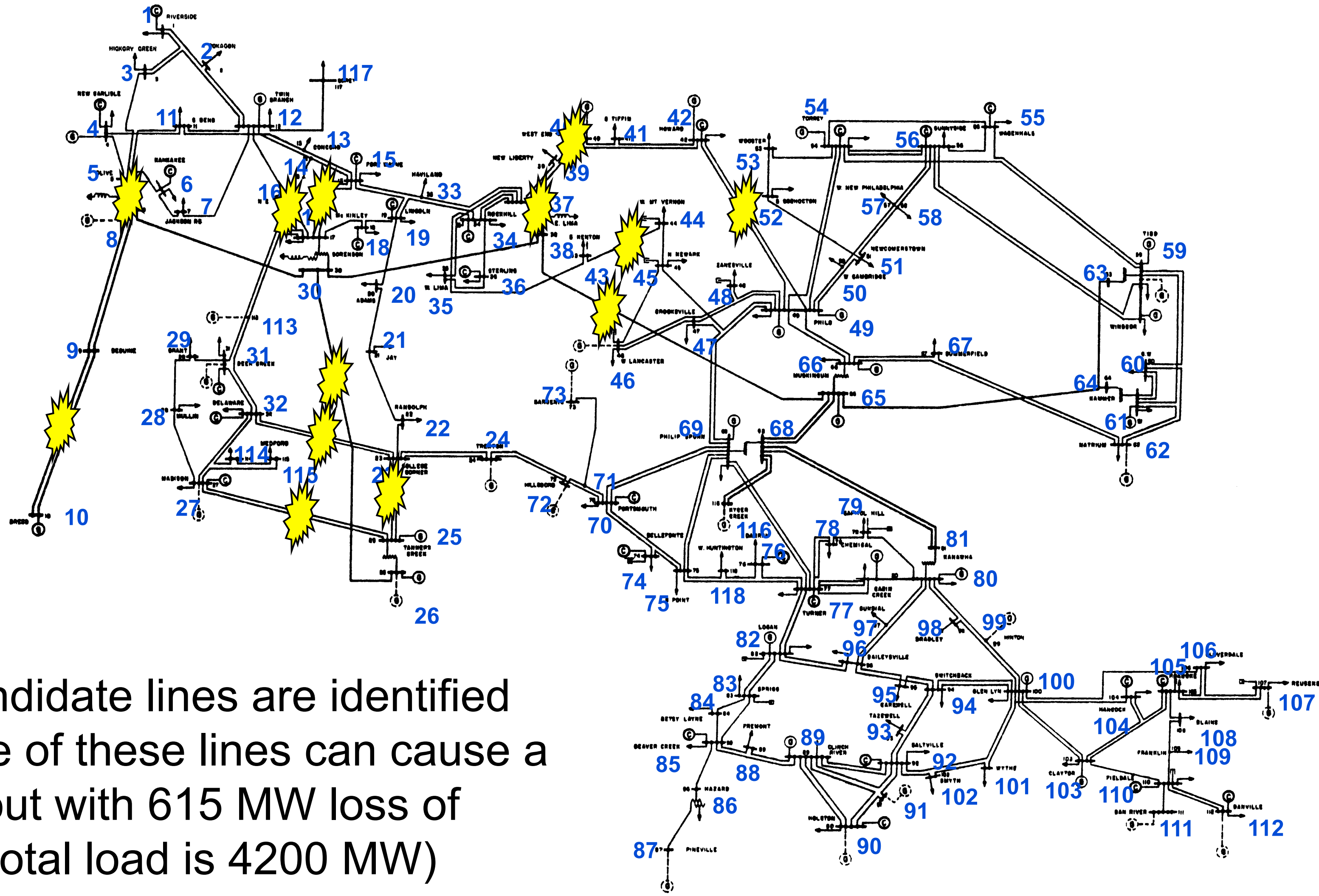
Relaxation works on small problems



IEEE 30-Bus System

- Four candidate lines identified.
- Two are sufficient to cause a blackout.
- Failure of these lines can cause a blackout with 843 MW loss out of a total load of 1655 MW).
- Solutions found using SNOPT.

…. but not on larger problems



IEEE 118 Bus System

- 13 candidate lines are identified
- Failure of these lines can cause a blackout with 615 MW loss of load (total load is 4200 MW)
- Better solutions exist

Computational Issues/Challenges

- The problem involves integer variables (need to employ relaxation).
- Nonlinear/nonconvex optimization problem leads to issues with local minima.
- Scalability – large scale systems pose a challenge due to increased computational burden and nonlinear optimization.
- The final solution and convergence is sensitive to initial conditions.

Exploiting the combinatorial structure

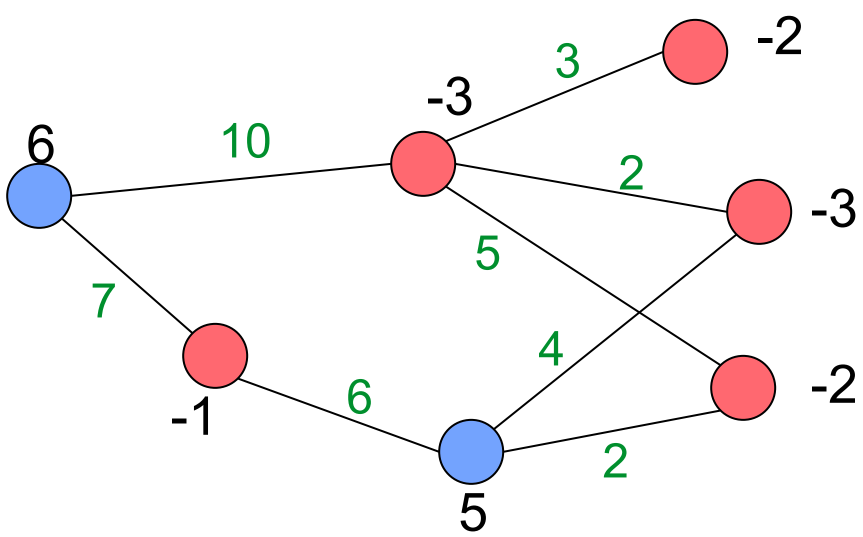
- Key new observation: *The **Jacobian matrix**, which characterizes the feasibility boundary, has the same structure as the **Laplacian matrix** in spectral graph theory.*
- Theoretical implications:
  - System is split into load-rich and generation-rich regions.
  - There is at least one saturated line from the generation rich region to the load rich region.
  - The size of the blackout can be approximated by the generation/load mismatch in one region and the capacity of edges in between.
- Practical application:
  - We can exploit the combinatorial structure to solve the vulnerability analysis problem.

Take 2: Vulnerability analysis as a combinatorial problem

Given a graph $G=(V,E)$ with weights on its vertices

- positive for generation,
- negative for loads,

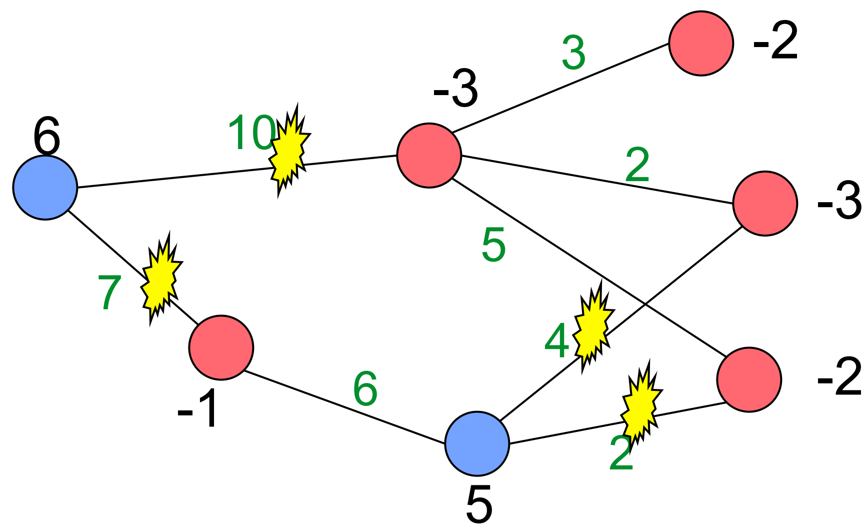
find a partition of V into two loosely connected regions with a significant load / generation mismatch.



Flow between the load-rich and generation-rich regions

- According to the theory, at least one line between the two regions is saturated in the direction from the generation rich region to the load rich region.
- Flow between the two regions can be bounded by the cumulative capacity of inter-region lines.
- Leads to two related problems:
  - Network inhibition problem (C. Phillips (SNL), Proceedings ACM Symposium on Theory of Computing, 1993)
  - Inhibiting bisection problem (Pinar, Fogel, Lesieutre, LBNL, 2007)

Network inhibition problem



k =0, max-flow= 11
k =1, max-flow= 7
k =2, max-flow= 5
k =3, max-flow=1

- Cut minimum number of lines so that max flow is below a specified bound.
- Shown to be NP-complete (Phillips 1991).
- Note that the classical min-cut problem is a special version of network inhibition, where max-flow is set to zero.

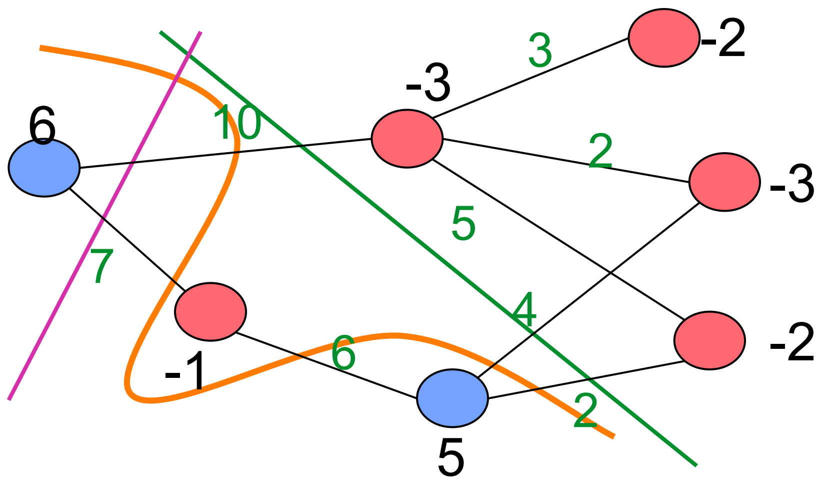
MILP formulation for network inhibition

- Cut minimum number of lines so that max-flow (min-cut) is below a specified bound.
- IP (Integer Programming) formulation for network inhibition:

$$
\begin{aligned}
\min \quad & \sum d_{ij} \\
s.t. \quad & \forall (v_i, v_j) \in E \quad
\begin{array}{l}
p_i - p_j - s_{ij} - d_{ij} \le 0 \\
p_i - p_j + s_{ij} + d_{ij} \ge 0
\end{array} \\
& \sum_{(v_i, v_j) \in E} c_{ij} s_{ij} \le B \\
& p_s = 0; \qquad p_t = 1 \\
& p_i, d_{ij}, s_{ij} \in \{0,1\};
\end{aligned}
$$

$$
p_i = \begin{cases} 0 & v_i \in S \\ 1 & v_i \in T \end{cases}
\qquad
d_{ij} = \begin{cases} 1 & \text{if } e_{ij} \text{ is cut.} \\ 0 & \text{otherwise} \end{cases}
\qquad
s_{ij} = \begin{cases} 1 & d_{ij} = 0 \wedge p_i \ne p_j \\ 0 & \text{otherwise} \end{cases}
$$

Inhibiting bisection problem



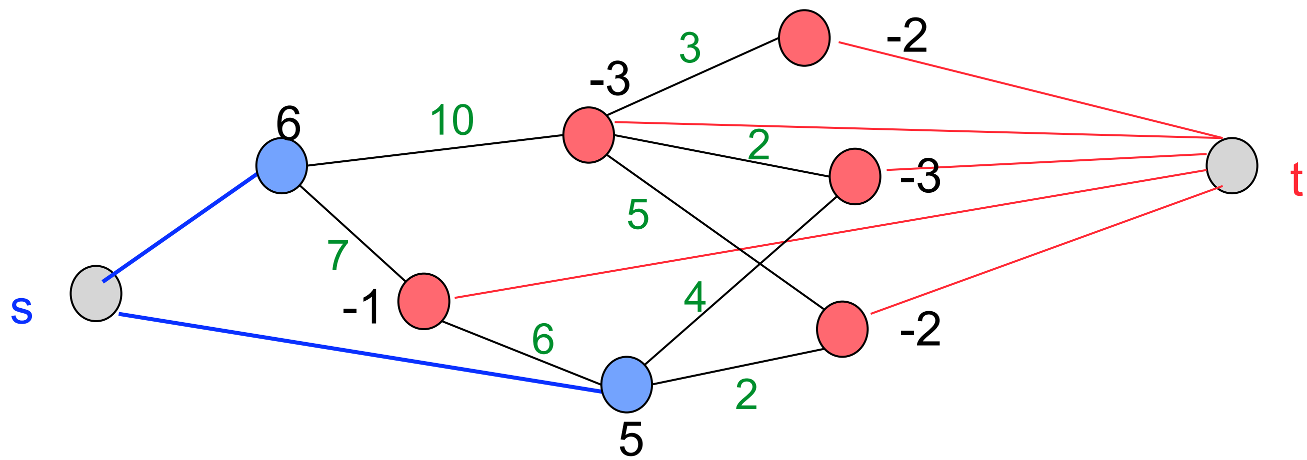
- Divide graph into two parts (bisection) so that
  - load/generation mismatch is maximum.
  - cutsize is minimum.

imbalance= 6; cutsize=2

imbalance=10; cutsize=3

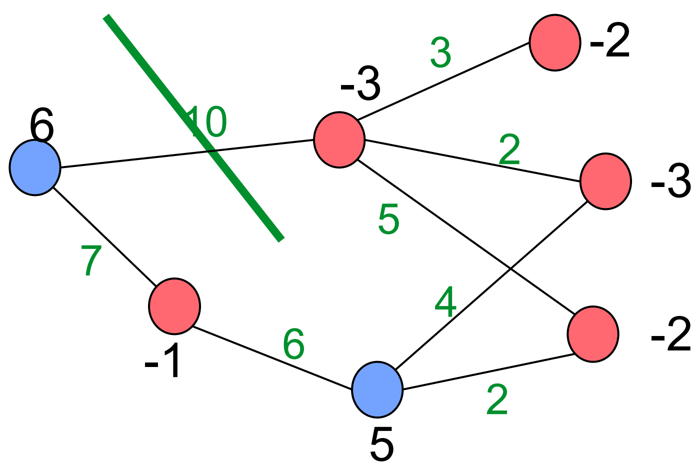
imbalance=11; cutsize=5

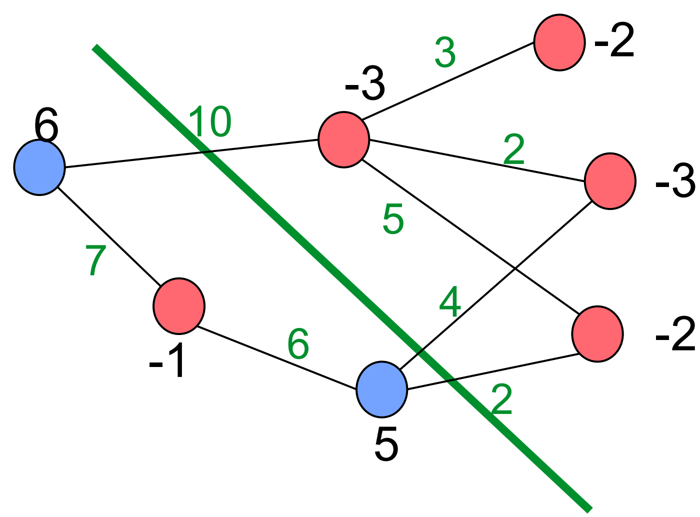
Solving the inhibiting cut problem



- Goal: minimize α (cutsize) - (1- α) imbalance
  - α is the relative importance of cutsize compared to imbalance.
- Solution: use a standard min-cut algorithm.
- Min-cut gives an ***optimal*** solution to the inhibiting bisection problem.

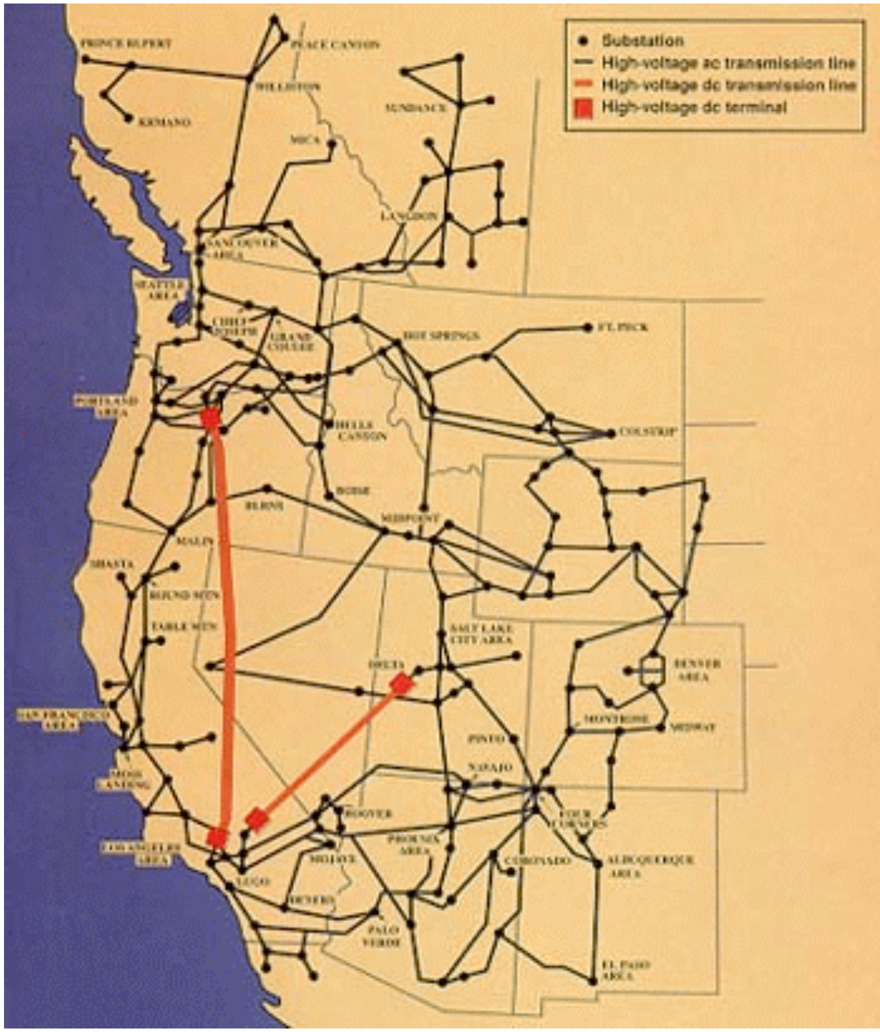
Comparison of combinatorial models for vulnerability analysis





Network inhibition	Inhibiting bisection
NP-complete	Polynomial-time versions available
Accurate formulation of the problem	Approximation to the real problem
Detects specific vulnerabilities	Detects groups of vulnerabilities
Better control for the analysis, there is a solution for any number of lines	Loose control; there are jumps in cutsizes

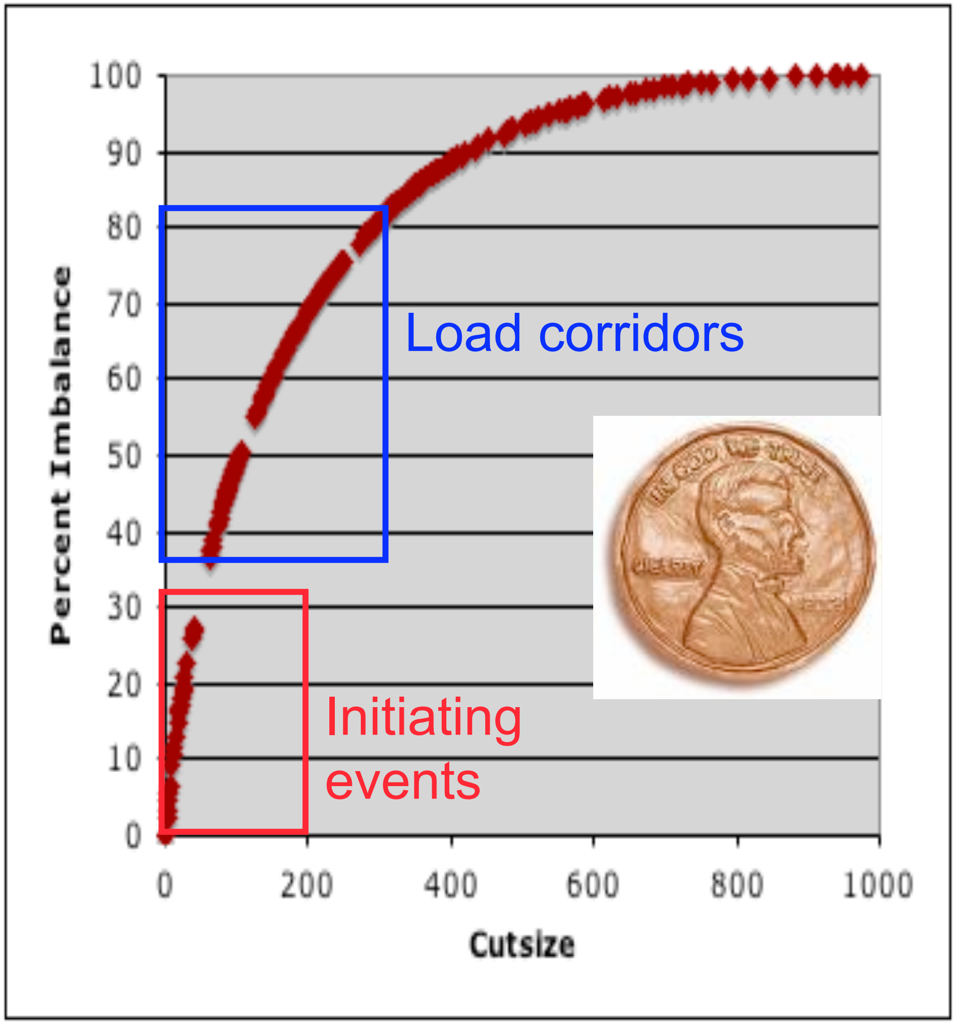
Take 3: Inhibiting bisection formulation



http://uwadmnweb.uwyo.edu/infotech/internet2/desc3.htm

- Simplified model for Western states
- 13,374 nodes and 16,520 lines.
- Complete analysis used Goldberg's min-cut solver
- Checked results with PICO, a massively parallel integer programming solver, developed by Phillips et al. at Sandia National Laboratories

Inhibiting bisection results



- Goldberg's min-cut solver takes minutes on standard desktop computer
- Solutions with small cutsize can be used to detect initiating events and groups of vulnerabilities
- Solutions with medium cutsize reveal load corridors.

Conclusions and future work

- Vulnerability analysis of a power system can be studied as a mixed integer nonlinear programming problem.
- Special structure of an optimal solution to the MINLP formulation can be exploited for a computationally easier approach.
- Our combinatorial techniques can analyze vulnerabilities of large systems in a short amount of time.
- Next goal: include vulnerability analysis as a component in decision support and policy making
- Many other applications of networks
  - Environmental management: exploit fracture networks for subsurface flows
  - Regulate pathways in biological networks
  - Transportation networks
  - Gas, water distribution networks

Questions?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Ace CANDLE Co.
SCHWADRON
"THE GOOD NEWS IS WE MAKE RECORD SALES EVERY TIME THERE'S A BLACKOUT."

Appendices

A: Power Flow Equations

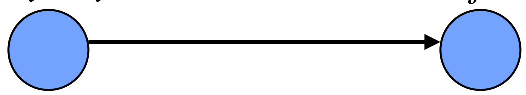
B: Spectral Graph Theory

C: Combinatorial Formulations

A: Power Flow Equations

Power Flow Equations

(V_i, θ_i) (V_j, θ_j)



$B_{ij} V_i V_j \sin(\theta_i - \theta_j)$ Active power

$B_{ij} V_i V_j \cos(\theta_i - \theta_j) + V_i^2$ Reactive power

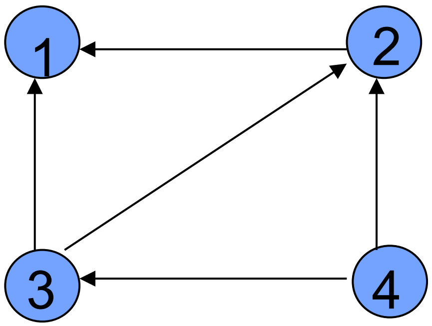
$$\frac{-\pi}{2} \le \theta_i - \theta_j \le \frac{\pi}{2}$$

$$V_l \le V \le V_u$$

V: voltage
θ: phase angle
B: susceptance

- Traditional graph algorithms are not directly applicable.
  - Nonlinearity makes use of traditional graph models difficult.
  - Flow is governed by variables on vertices.

Power Flow Equations



$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Active $$p = A^T D(e^{|A|\ln V}) B \sin(A\theta)$$

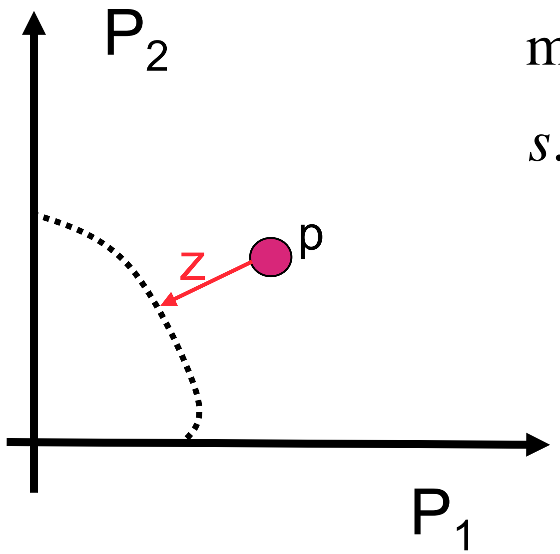
Reactive $$q = -|A^T| D(e^{|A|\ln V}) B \cos(A\theta) + V^2 D(A^T BA)$$

$$-\pi/2 \le A\theta \le \pi/2; \qquad V_L \le V \le V_U$$

- Simplified model for power flow:
  - Fix voltages at 1.
  - Work only on active power.

$$F(A,\theta,p) = A^T B \sin(A\theta) - p = 0$$

Measuring the severity of a blackout



$$\min_{\theta,z} |-z_g|$$

Minimum load shed

$$s.t. \quad F(A,\theta,p+z)=0$$

Feasible power flow

$$-\pi/2 \le A\theta \le \pi/2$$

$$0 \le p_g + z_g \le p_g$$

Generators remain as generators.

$$p_l \le p_l + z_l \le 0$$

Loads remain as loads.

KKTconditions

$$\begin{pmatrix}-e\\0\end{pmatrix} + \lambda^T \frac{\partial F}{\partial z} + \begin{pmatrix}\mu_4-\mu_3\\ \mu_2-\mu_1\end{pmatrix} = 0$$

$$\lambda^T \frac{\partial F}{\partial \theta} + A^T(\mu_6-\mu_5)=0$$

$$\mu_1 z_l = 0; \quad \mu_2(p_l+z_l)=0;$$

$$\mu_4 z_g = 0; \quad \mu_3(p_g+z_g)=0;$$

$$\mu_5(\pi/2+A\theta)=0; \quad \mu_6(\pi/2-A\theta)=0;$$

$$\mu_1,\ldots,\mu_6 \ge 0$$

B: Spectral Graph Theory

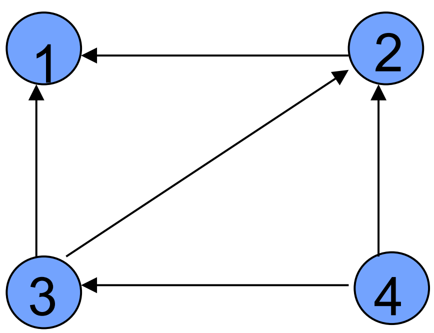
Feasibility boundary and spectral graph theory

On the boundary of feasibility, the power-flow Jacobian, *J*, will have its second singular vector, when inequality constraints are inactive.

$$\frac{\partial F}{\partial \theta} = J = A^T BD((1-\gamma)\cos(A\theta)A$$

$$Jw = 0; \qquad w^T e = 0; w^T w = 1$$

J has the same structure as Laplacian in spectral graph theory.



$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Longrightarrow A^T DA = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Feasibility boundary and spectral graph theory

Theorem: The number of singular vectors of the Laplacian is equal to the number of connected components of its graph.

Corollary: At the boundary of feasibility the power grid is divided into two regions by lines that are cut or saturated.

$$\frac{\partial F}{\partial \theta} = J = P\begin{pmatrix} J_{11} & 0 \\ 0 & J_{22} \end{pmatrix} P^T = A^T BD((1-\gamma)\cos(A\theta)A$$

Impact: Setting Lagrangian multipliers to 0 yields direct transformation of our MINLP formulation to a combinatorial problem.

$$J\lambda + A^T(\mu_6 - \mu_5) = 0$$

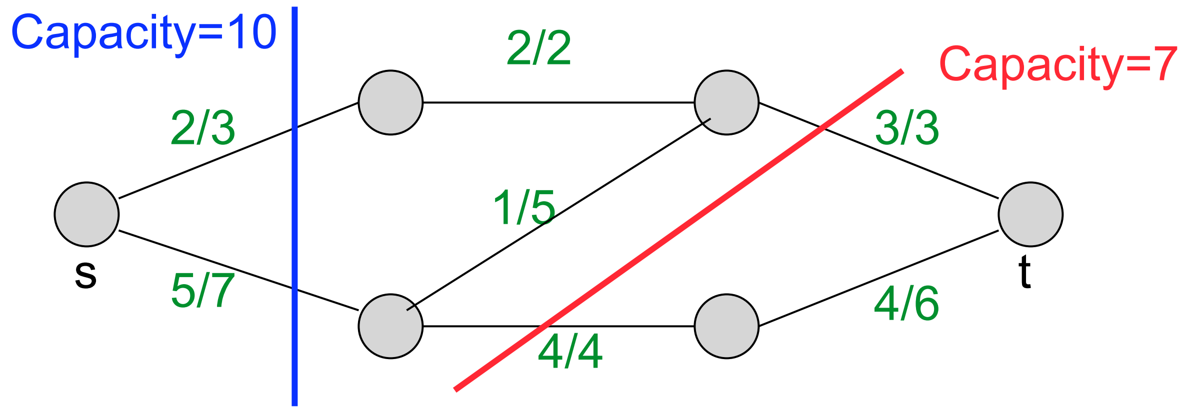
Structure of an optimal solution: load and generation-rich regions

Analysis of the KKT conditions reveals a special structure of an optimal solution.

$$\begin{pmatrix} -e \\ 0 \end{pmatrix} - \begin{pmatrix} \lambda_g \\ \lambda_l \end{pmatrix} + \begin{pmatrix} \mu_4 - \mu_3 \\ \mu_2 - \mu_1 \end{pmatrix} = 0$$

- The system is decomposed into two regions.
  - Generation-rich region $\lambda_i < 0$
    - No decrease in loads, generation can be shed.
  - Load-rich region $\lambda_i \geq 0$
    - No decrease in generation, loads can be shed.

Maximum-flow and minimum cut



- Given a graph, with capacities on edges, a source vertex, s, and a terminal vertex, t, the objective is to push as much flow as possible from the source to the terminal.
- Cut is a bipartitioning of the vertices into S and T, so that s in S and t in T.
  - Capacity of a cut is the cumulative capacity of edges between S and T.
  - Min-cut is a cut with minimum capacity.
- Volume of max-flow = capacity of a min-cut.

C: Combinatorial Formulations

MILP formulation for network inhibition

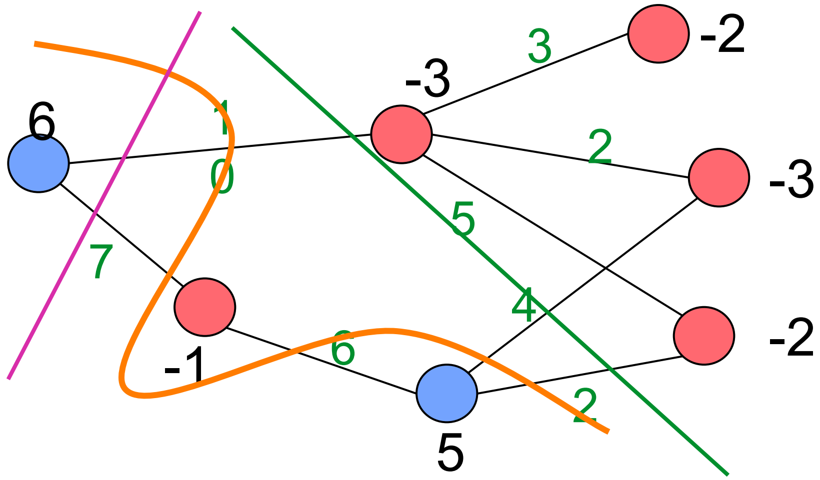
- Cut minimum number of lines so that max-flow (min-cut) is no more than a specified bound.
- IP formulation for min-cut:

$$
\begin{aligned}
&\min \quad \sum c_{ij} s_{ij} \\
&s.t. \quad \forall (v_i, v_j) \in E \quad \begin{array}{l} p_i - p_j - s_{ij} \le 0 \\ p_i - p_j + s_{ij} \ge 0 \end{array} \\
&\qquad p_s = 0; \qquad p_t = 1 \\
&\qquad p_i, s_{ij} \in \{0,1\};
\end{aligned}
$$

  - $c_{ij} > 0$ is the capacity of edge e_{ij}.
  - $$p_i = \begin{cases} 0 & v_i \in S \\ 1 & v_i \in T \end{cases} \qquad s_{ij} = \begin{cases} 1 & p_i \neq p_j \\ 0 & otherwise \end{cases}$$

Inhibiting bisection

Minimize *x* (cutsize)- imbalance



- $x>4$ $2x-6$
- $0.5<x<4$ $3x-10$
- $x<0.5$ $5x-11$

Inhibiting bisection (constrained version)

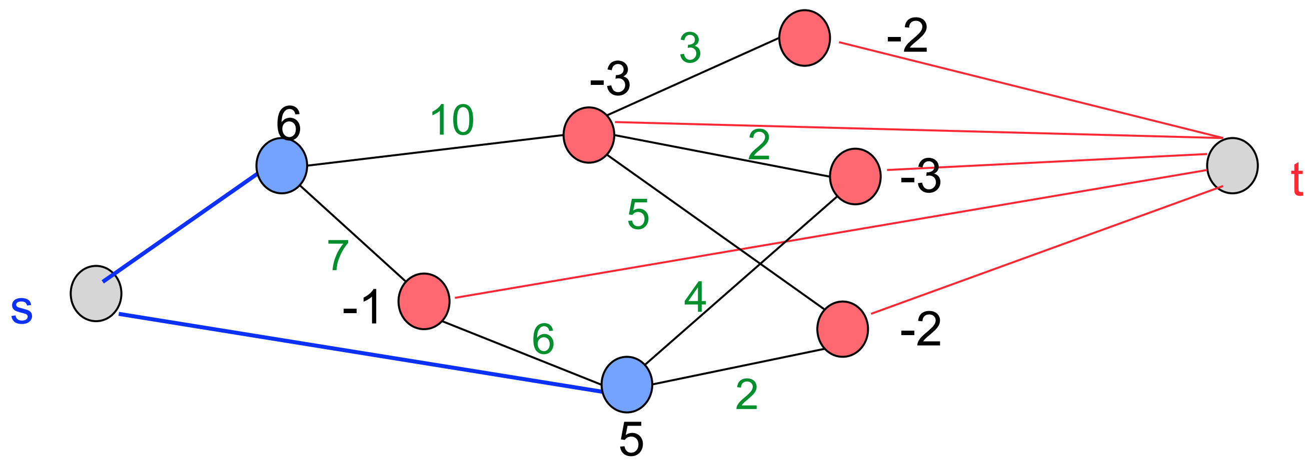
- Given a graph *G=(V,E)* with weights w_i on its vertices,
  - $w_i > 0$ for generation,
  - $w_i \leq 0$ for consumption,

find a bipartition of *V* into *S* and *T* with maximum imbalance, where the cutsize is below a specified threshold.

  - Imbalance: $\sum_{v_i \in S} w_i$
  - Cutsize: $\left|\{(v_i, v_j) \in E, v_i \in S, v_j \in T\}\right|$

- NP-complete.
  - Reduction from graph bisection problem.
- Allowing trade-off allows a polynomial time solution
  - Min α (cutsize) - (1- α) imbalance

Solving the inhibiting cut problem



- Goal: minimize α (cutsize) - (1- α) imbalance
  - α is the relative importance of cutsize compared to imbalance.
- Solution: use a minimum-cut algorithm.
- Method: use balance edges to connect each generation (load) vertex to s(t).
  - If a generator (load) is in part T(S),
    - its balance edge will be cut, and imbalance metric will decrease by the vertex weight.
    - Assign this change as the capacity of the balance edge.
  - Other edges affect the cutsize.
    - Their weights are assigned as α.
- Minimum cut gives an optimal solution to the inhibiting bisection problem.